# Computation of Resonant Frequencies and Quality Factors of Open Dielectric Resonators by a Combination of the Finite-Difference Time-Domain (FDTD) and Prony's Methods

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*Abstract*—**Open dielectric resonators using the FDTD method are analyzed. Resonant frequencies and quality factors are calculated using Prony's method instead of the classical fast Fourier transform. In this way, reductions of up to two orders of magnitude are achieved in computation time. The results obtained are in good agreement with those reported by other authors.**

## I. Introduction

In MOST practical applications, dielectric resonators (DR's) are part of complex structures which normally include mechanisms for tuning and coupling with external circuits; hence in order to study DR's it is necessary to use purely numerical methods. A DR in a circuit is characterized by three parameters: resonant frequency, quality factor and coupling factor. The finite-difference time-domain (FDTD) method [1] has already been employed in the characterization of DR's enclosed in cavities [2], [3], so it should allow these three parameters to be determined. Since it is a time-domain method, it must be used in combination with another technique which, starting from the time response obtained with FDTD, will provide us with the resonant frequencies and quality factors.

In previous applications of the FDTD method to DR's and other eigenvalue problems [2]–[4], the step from the time-domain response to the frequency-domain response has been computed using the fast Fourier transform (FFT). As a result, a large number of time samples was needed for a reliable characterization of the resonance frequency of the structure, which means unduly high computation time.

This letter proposes Prony's method as an alternative to FFT since it is capable of obtaining resonant frequencies and quality factors from shorter time-domain responses, thus reducing CPU time and memory requirements.

## II. Theory

Results in the frequency domain are usually obtained by recording the time-domain response at selected observation points of the FDTD mesh and applying the FFT algorithm. Then, the resonant frequencies are calculated from the local maxima of the spectrum. There are, however, several inherent performance limitations in the FFT approach. The main limitation is that of the frequency resolution $\Delta f$, which is roughly the reciprocal of observation time, i.e., $\Delta f = 1/N\Delta t$ where $\Delta t$ is the time step and $N$ the total number of iterations used in the FDTD method. A second limitation arises from the windowing of the time-domain data because the FDTD response is truncated in time. This has the effect of viewing the true time-domain response through a rectangular window of duration $T = N\Delta t$. In the frequency domain this windowing is translated into the convolution of the true spectrum with the function $\sin(f)/f$. This convolution widens the peaks in the spectrum response, the whole spectrum is distorted and some weak signal spectral responses can be masked. Distortion can be reduced and spectral resolution increased only by making the duration of the window longer, that is by increasing simulation time, which in turn leads to an undue increase in CPU time and memory requirements.

Attempts to alleviate the limitations of the FFT approach have led to numerous alternative spectral estimation procedures being proposed in applications such as speech processing. For our work, Prony's method was chosen because it is particularly suitable for the calculation of resonant frequencies and quality factors. Prony's method is a technique for modeling sampled data as a linear combination of complex exponentials. Although it is not a spectral estimation technique in the usual sense, it is closely related to the least squares linear prediction methods used for parametric spectral estimation [5]. The FDTD time-domain response of an eigenvalue problem can be expressed in terms of a superposition of the resonant modes

$$S(n\Delta t) = \sum_{i=1}^{p} A_i \exp\left((\alpha_i + j2\pi f_i)n\Delta t\right) \quad \text{for } n = 0, \cdots, N-1, \qquad (1)$$

where $S$ is one of the six electromagnetic field components, $A_i$ is the complex modal amplitude, $\alpha_i$ the damping factor and $f_i$ the resonant frequency of the $i$th resonant mode, and $p$ is the order of the model (twice the number of resonant modes). The direct solution of (1) is a difficult nonlinear least squares problem. An alternative solution is based on Prony's method, which solves two sequential sets of linear

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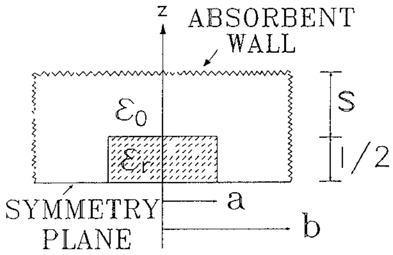


Fig. 1. Isolated cylindrical dielectric resonator.

equations with an intermediate polynomial rooting step that concentrates the nonlinearity of the problem. This technique shows good resolution with relatively short data sequences and presents no problems of windowing. The main difficulty with Prony's method lies in the determination of the order of the model, $p$. If the value of $p$ is very low (less than the number of excited resonant modes), the spectral resolution is poor. On the other hand, if the value of $p$ is very high, spurious modes appear. One way of distinguishing between real and spurious modes is to apply Prony's method with the time sequence in reverse order; in this way, real modes appear with positive damping factors ($\alpha_i > 0$), but spurious modes with negative damping factors ($\alpha_i < 0$) [6]. Before Prony's method is applied, the time-domain response is submitted to preprocessing that involves low pass filtering in order to limit the number of resonant modes and, therefore, the number of parameters to calculate; the signal is then decimated since the FDTD method usually gives rise to oversampled signals.

Once the damping factor and the resonant frequency have been determined, the quality factor can easily be obtained as

$$Q_i = \left| \frac{\pi f_i}{\alpha_i} \right|. \tag{2}$$

## III. Results

To illustrate how using Prony's method instead of the FFT algorithm can reduce the number of iterations in the FDTD method, we studied an isolated cylindrical dielectric resonator with a radius $a = 5.25$ mm, height $l = 4.6$ mm, and dielectric constant $\epsilon_r = 38$ shown in Fig. 1. The resonant frequency and quality factor of the first three modes $TE_{0n}$ of the mentioned DR are represented in Fig. 2. The absorbing conditions employed are based on a parabolic interpolation [7] and have been placed at a distance 5 times the dimensions of the resonator ($b = 5a$ and $s = 2l$). It can be seen that with the FFT algorithm at least $2^{17}$ iterations are necessary to achieve convergence in the resonant frequency and quality factor, whereas with Prony's method 3000 iterations are sufficient. The reduction can be even greater in the case of modes with a higher quality factor or with resonant frequencies that are very close to each other. The size of the space mesh employed in the FDTD method was $\Delta r = 0.29167$ mm, and $\Delta z = 0.2875$ mm, and the time step $\Delta t = 0.47950$ ps. In the model used to obtain the results of Fig. 2, the order was $p = 10$ and the decimation factor was 50.

In order to verify the accuracy of the FDTD-Prony method in characterizing open DR's, the results obtained for the first six modes of the resonator of Fig. 1 have been compared (Tables I-A and I-B) with those available in the literature [8]–[10]. It can be seen that the agreement is good. The results for the FDTD-Prony method were obtained using the same conditions as in Fig. 2.

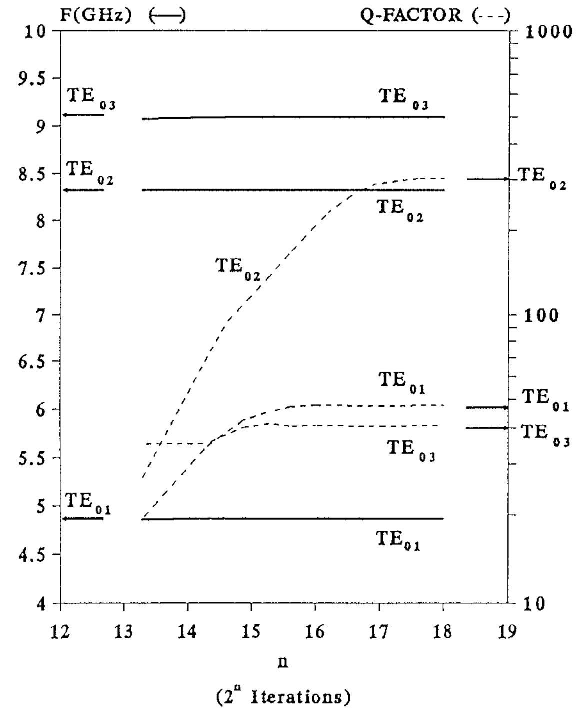


Fig. 2. Convergence of the resonant frequencies and $Q$-factors of the three lowest $TE_{0n}$ modes of an isolated DR with the number of FDTD iterations. Lines: FDTD–FFT method (———) frequencies, (- - -) $Q$-factors. Arrows: FDTD–Prony method with 3000 FDTD iterations. FDTD parameters: $\Delta r = 0.29167$ mm, $\Delta z = 0.2875$ mm, $\Delta t = 0.47950$ ps.

TABLE I-A
Comparison of the Resonant Frequencies and $Q$-Factors of the Six Lowest Modes of an Isolated DR Obtained by Various Methods
Axisymmetric Modes

| | $TE_{01}$ | | $TM_{01}$ | | $TE_{02}$ | |
|---|---|---|---|---|---|---|
| | $F$(GHz) | $Q$ | $F$(GHz) | $Q$ | $F$(GHz) | $Q$ |
| Moment Method [9] | 4.829 | 45.8 | 7.524 | 76.8 | | |
| Null-Field Method [8] | 4.8604 | 40.819 | 7.5384 | 76.921 | 8.3311 | 301.02 |
| Measured [10] | 4.85 | 51 | 7 60 | 86 | | |
| Present Method | 4.862 | 47 | 7.524 | 71 | 8.320 | 302 |

## IV. Conclusion

Combining the FDTD method and Prony's method allows the resonant frequency values and quality factors of DR's to be obtained with an accuracy similar to that of other rigorous methods. Moreover, the use of Prony's method rather than FFT considerably reduces CPU time. It has been shown that a reduction of two orders of magnitude can be obtained even for simple cases (resonant frequencies which are not very close and quality factors that are not very high).

TABLE I-B
COMPARISON OF THE RESONANT FREQUENCIES AND $Q$-FACTORS OF THE SIX LOWEST MODES OF AN ISOLATED DR OBTAINED BY VARIOUS METHODS
HYBRID MODES

| | $HEM_{11}$ | | $HEM_{12}$ | | $HEM_{21}$ | |
|---|---|---|---|---|---|---|
| | $F$(GHz) | $Q$ | $F$(GHz) | $Q$ | $F$(GHz) | $Q$ |
| Moment Method [9] | 6.333 | 30.7 | 6.638 | 52.1 | 7.752 | 327.1 |
| Null-Field Method [8] | 6.3450 | 30.853 | 6.6520 | 50.316 | 7.7621 | 337.66 |
| Measured [10] | | | 6.64 | 64 | 7.81 | 204 |
| Present Method | 6.344 | 31 | 6.648 | 46 | 7.751 | 324 |

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